

REVISIONS JUIN 2012

Algèbre - Exercices

Chapitre 4: les inéquations

Détermine si le nombre -2 est solution des inéquations suivantes:

1) $3x + 4 \leq -2$

$$3 \cdot (-2) + 4 \not\leq -2$$

$$-6 + 4 \not\leq -2$$

$$-2 \leq -2$$

-2 est solution de l'inéquation.

2) $\frac{2x-1}{2} \geq \frac{5-3x}{3}$

$$\frac{2 \cdot (-2) - 1}{2} \not\geq \frac{5 - 3 \cdot (-2)}{3}$$

$$\frac{-4 - 1}{2} \not\geq \frac{5 + 6}{3}$$

$$\frac{-3}{2} < \frac{11}{3}$$

-2 n'est pas solution de l'inéquation.

Résous les inéquations suivantes:

(--- = vert ; — = rouge)

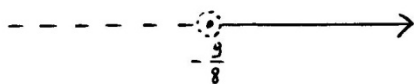
1) $-3x - 7 \geq 5x + 2$

$$\Leftrightarrow -3x - 5x \geq 2 + 7$$

$$\Leftrightarrow -8x \geq 9$$

$$\Leftrightarrow x \leq -\frac{9}{8}$$

$$S =]-\infty; -\frac{9}{8}]$$



2) $3(2x-4) \leq -5(4x+1) - 5x$

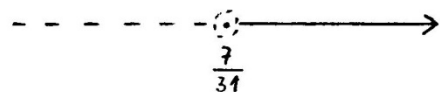
$$\Leftrightarrow 6x - 12 \leq -20x - 5 - 5x$$

$$\Leftrightarrow 6x + 20x + 5x \leq -5 + 12$$

$$\Leftrightarrow 31x \leq 7$$

$$\Leftrightarrow x \leq \frac{7}{31}$$

$$S =]-\infty; \frac{7}{31}]$$



$$3) \frac{4x-1}{5} - \frac{2x+3}{3} < 0$$

$$\Leftrightarrow \frac{3(4x-1) - 5(2x+3)}{15} < 0$$

$$\Leftrightarrow 12x - 3 - 10x - 15 < 0$$

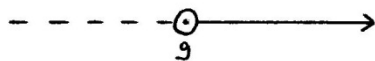
$$\Leftrightarrow 12x - 10x < 3 + 15$$

$$\Leftrightarrow 2x < 18$$

$$\Leftrightarrow x < \frac{18}{2}$$

$$\Leftrightarrow x < 9$$

$$S =]-\infty; 9[$$



$$4) (2x+1)^2 > (x-2)(4x+3) - 7$$

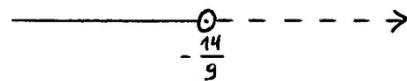
$$\Leftrightarrow 4x^2 + 4x + 1 > 4x^2 + 3x - 8x - 6 - 7$$

$$\Leftrightarrow 4x^2 + 4x - 4x^2 - 3x + 8x > -6 - 7 - 1$$

$$\Leftrightarrow 9x > -14$$

$$\Leftrightarrow x > -\frac{14}{9}$$

$$S =]-\frac{14}{9}; +\infty[$$



Chapitre 5 : les polynômes

Calcule la valeur numérique des polynômes pour $x = -1$ et pour $x = 2$:

$$P(x) = 3x^3 - 2x^2 + x - 5$$

$$P(-1) = 3 \cdot (-1)^3 - 2 \cdot (-1)^2 + (-1) - 5 = 3 \cdot (-1) - 2 \cdot 1 - 1 - 5 = -3 - 2 - 1 - 5 = -11$$

$$P(2) = 3 \cdot 2^3 - 2 \cdot 2^2 + 2 - 5 = 3 \cdot 8 - 2 \cdot 4 + 2 - 5 = 24 - 8 + 2 - 5 = 13$$

$$Q(x) = -2x^2 + x^2 - 5x + 2 = -x^2 - 5x + 2$$

$$Q(-1) = -(-1)^2 - 5 \cdot (-1) + 2 = -1 + 5 + 2 = 6$$

$$Q(2) = -2^2 - 5 \cdot 2 + 2 = -4 - 10 + 2 = -12$$

On donne les polynômes suivants; ordonne-les et complète-les :

$$P(x) = 2x - 5x^2 + x^3 + 12 = x^3 - 5x^2 + 2x + 12$$

$$Q(x) = 16 - 8x + 3x^3 + 5x^4 = 5x^4 + 3x^3 + 0x^2 - 8x + 16$$

$$R(x) = -5x + 3x^2 - 2x^3 + 8 - x = -2x^3 + 3x^2 - 6x + 8$$

$$S(x) = 2x^2 - 3 = 2x^2 + 0x - 3$$

$$T(x) = -5 - 2x^2 + 3x = -2x^2 + 3x - 5$$

$$U(x) = x^3 + 5x - 3x^2 - 2 = x^3 - 3x^2 + 5x - 2$$

$$V(x) = x - 2$$

$$W(x) = x^5 - 3x^2 + 1 = x^5 + 0x^4 + 0x^3 - 3x^2 + 0x + 1$$

$$Y(x) = x - 1$$

$$Z(x) = 4x^3 - 15 - x = 4x^3 + 0x^2 - x - 15$$

Effectue en utilisant les dispositions pratiques:

$$\begin{array}{r}
 P(x) + Q(x) - R(x) \\
 P(x) \rightarrow x^3 - 5x^2 + 2x + 12 \\
 Q(x) \rightarrow 5x^4 + 3x^3 + 0x^2 - 8x + 16 \\
 + \quad - R(x) \rightarrow \underline{2x^3 - 3x^2 + 6x - 8} \\
 \hline
 5x^4 + 6x^3 - 8x^2 + 0x + 20
 \end{array}$$

$$\begin{array}{r}
 S(x) \cdot T(x) \\
 S(x) \rightarrow 2x^2 + 0x - 3 \\
 \cdot \quad T(x) \rightarrow \underline{-2x^2 + 3x - 5} \\
 \hline
 -10x^2 - 0x + 15 \\
 6x^3 + 0x^2 - 9x \\
 + \quad \underline{-4x^4 - 0x^3 + 6x^2} \\
 \hline
 -4x^4 + 6x^3 - 4x^2 - 9x + 15
 \end{array}$$

$$\begin{array}{r}
 U(x) \cdot T(x) \\
 U(x) \rightarrow x^3 - 3x^2 + 5x - 2 \\
 \cdot \quad T(x) \rightarrow \underline{-2x^2 + 3x - 5} \\
 \hline
 -5x^3 + 15x^2 - 25x + 10 \\
 3x^4 - 9x^3 + 15x^2 - 6x \\
 + \quad \underline{-2x^5 + 6x^4 - 10x^3 + 4x^2} \\
 \hline
 -2x^5 + 9x^4 - 24x^3 + 34x^2 - 31x + 10
 \end{array}$$

$$U(x) : V(x)$$

$$\begin{array}{r|rrrr|r} & 1 & -3 & 5 & & -2 \\ 2 & \downarrow & & & & \\ \hline & 1 & -1 & 3 & & 4 \end{array}$$

Division par $(x-2) \Rightarrow$ Horner

$$x^3 - 3x^2 + 5x - 2 = (x-2)(x^2 - x + 3) + 4$$

$$W(x) : Y(x)$$

$$\begin{array}{r|rrrrr|r} & 1 & 0 & 0 & -3 & 0 & 1 \\ 1 & \downarrow & & & & & \\ \hline & 1 & 1 & 1 & -2 & -2 & -1 \end{array}$$

Division par $(x-1) \Rightarrow$ Horner

$$x^5 - 3x^2 + 1 = (x-1)(x^4 + x^3 + x^2 - 2x - 2) - 1$$

$$Z(x) : T(x)$$

$$\begin{array}{r} Z(x) \rightarrow \begin{array}{l} (4x^3) + 0x^2 - x - 15 \\ - 4x^3 + 6x^2 - 10x \\ \hline 6x^2 - 11x - 15 \\ - 6x^2 + 9x - 15 \\ \hline 2x^2 - 2x - 30 \\ \hline \text{reste} \end{array} \quad \begin{array}{r} \overline{(-2x^2 + 3x - 5)} \\ - 2x - 3 \\ \hline \text{quotient} \end{array} \end{array}$$

$$4x^3 - x - 15 = (-2x^2 + 3x - 5)(-2x - 3) + (-2x - 30)$$

Chapitre 6 : la factorisation

Factorise les expressions suivantes :

$$1) 12a^2bc^2 + 18a^2b^2c^3 = 6a^2bc^2(2 + 3bc)$$

$$2) a^5b^3 + a^4b^7 - a^2b^2 = a^2b^2(a^3b + a^2b^5 - 1)$$

$$\begin{aligned} 3) (x+1)(x+5) + 7(x+1) &= (x+1)[(x+5) + 7] \\ &= (x+1)(x+12) \end{aligned}$$

$$\begin{aligned} 4) (x+2)(3x+4) + (x+2)(x-3) &= (x+2)[(3x+4) + (x-3)] \\ &= (x+2)(4x+1) \end{aligned}$$

$$\begin{aligned} 5) (x-3)(4x+9) - 5(3-x) &= (x-3)(4x+9) + 5(x-3) \\ &= (x-3)[(4x+9) + 5] \\ &= (x-3)(4x+14) \\ &= 2(x-3)(2x+7) \end{aligned}$$

$$\begin{aligned}
 6) (2x+4)(x-1) + (x-7)(1-x) &= (2x+4)(x-1) - (x-7)(x-1) \\
 &= (x-1)[(2x+4) - (x-7)] \\
 &= (x-1)(2x+4-x+7) \\
 &= (x-1)(x+11)
 \end{aligned}$$

$$7) 25x^2 - 9 = (5x-3)(5x+3)$$

$$\begin{aligned}
 8) 36x^2 - (2x+5)^2 &= [6x - (2x+5)][6x + (2x+5)] \\
 &= (6x-2x-5)(6x+2x+5) \\
 &= (4x-5)(8x+5)
 \end{aligned}$$

$$\begin{aligned}
 9) (2x-3)^2 - (3x+5)^2 &= [(2x-3) - (3x+5)][(2x-3) + (3x+5)] \\
 &= (2x-3-3x-5)(2x-3+3x+5) \\
 &= (-x-8)(5x+2)
 \end{aligned}$$

$$10) x^2 + 4x + 4 = (x+2)^2$$

$$11) x^2 - 22x + 121 = (x-11)^2$$

$$12) 9x^2 + 12x + 4 = (3x+2)^2$$

$$13) 25x^2 + 30x + 9 = (5x+3)^2$$

$$\begin{aligned}
 14) 2x^4 + 2x^3 + 3x + 3 &= (2x^4 + 2x^3) + (3x + 3) \\
 &= 2x^3(x+1) + 3(x+1) \\
 &= (x+1)(2x^3+3)
 \end{aligned}$$

$$\begin{aligned}
 15) 6x^4 - 3x^3 - 4x + 2 &= (6x^4 - 3x^3) - (4x - 2) \\
 &= 3x^3(2x-1) - 2(2x-1) \\
 &= (2x-1)(3x^3-2)
 \end{aligned}$$

$$16) 2x^2 + x - 10 = (x-2)(2x+5)$$

$$\text{div } 10 = \{\pm 1; \pm 2; \pm 5; \pm 10\}$$

$$P(2) = 2 \cdot 2^2 + 2 - 10 = 2 \cdot 4 + 2 - 10 = 8 + 2 - 10 = 0 \Rightarrow \text{div. par } (x-2)$$

2	2	1	-10
2	↓	4	10
2	2	5	0

$$17) -3x^2 + 2x + 1 = (x-1)(-3x-1)$$

$$\text{div } 1 = \{\pm 1\}$$

$$P(1) = -3 \cdot 1^2 + 2 \cdot 1 + 1 = -3 + 2 + 1 = 0 \Rightarrow \text{div. par } (x-1)$$

$$\begin{array}{c|cc|c} & -3 & 2 & 1 \\ 1 & \downarrow & -3 & -1 \\ \hline & -3 & -1 & 0 \end{array}$$

$$18) x^3 - 4x^2 + x + 6 = (x+1)(x^2 - 5x + 6) *$$

$$\text{div } 6 = \{\pm 1; \pm 2; \pm 3; \pm 6\}$$

$$P(-1) = (-1)^3 - 4 \cdot (-1)^2 + (-1) + 6 = -1 - 4 - 1 + 6 = 0 \Rightarrow \text{div. par } (x+1)$$

$$\begin{array}{c|ccc|c} & 1 & -4 & 1 & 6 \\ -1 & \downarrow & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$$* (x+1)(x^2 - 5x + 6) = (x+1)(x-2)(x-3)$$

$$\text{div } 6 = \{\pm 1; \pm 2; \pm 3; \pm 6\}$$

$$P(2) = 2^2 - 5 \cdot 2 + 6 = 4 - 10 + 6 = 0 \Rightarrow \text{div. par } (x-2)$$

$$\begin{array}{c|cc|c} & 1 & -5 & 6 \\ 2 & \downarrow & 2 & -6 \\ \hline & 1 & -3 & 0 \end{array}$$

Chapitre 7: les équations réductibles au premier degré

Résous les équations suivantes:

$$1) (x+2)(x-1) = 0$$

$$\Leftrightarrow x+2 = 0 \text{ ou } x-1 = 0$$

$$\Leftrightarrow x = -2 \text{ ou } x = 1$$

$$S = \{-2; 1\}$$

$$2) 12x^2 = 4x$$

$$\Leftrightarrow 12x^2 - 4x = 0$$

$$\Leftrightarrow 4x(3x-1) = 0$$

$$\Leftrightarrow 4x = 0 \text{ ou } 3x-1 = 0$$

$$\Leftrightarrow x = 0 \text{ ou } x = \frac{1}{3}$$

$$S = \left\{0; \frac{1}{3}\right\}$$

$$3) 16x^2 - 25 = 0$$

$$\Leftrightarrow (4x-5)(4x+5) = 0$$

$$\Leftrightarrow 4x-5 = 0 \text{ ou } 4x+5 = 0$$

$$\Leftrightarrow x = \frac{5}{4} \text{ ou } x = -\frac{5}{4}$$

$$S = \left\{ -\frac{5}{4}; \frac{5}{4} \right\}$$

$$5) 2x(2x-3)(4x-5)^2 = 0$$

$$\Leftrightarrow 2x = 0 \text{ ou } 2x-3 = 0 \text{ ou } 4x-5 = 0$$

$$\Leftrightarrow x = 0 \text{ ou } x = \frac{3}{2} \text{ ou } x = \frac{5}{4}$$

$$S = \left\{ 0; \frac{5}{4}; \frac{3}{2} \right\}$$

$$7) (3x-1)^2 = (2x+5)^2$$

$$\Leftrightarrow (3x-1)^2 - (2x+5)^2 = 0$$

$$\Leftrightarrow [(3x-1) - (2x+5)][(3x-1) + (2x+5)] = 0$$

$$\Leftrightarrow (3x-1-2x-5)(3x-1+2x+5) = 0$$

$$\Leftrightarrow (x-6)(5x+4) = 0$$

$$\Leftrightarrow x-6 = 0 \text{ ou } 5x+4 = 0$$

$$\Leftrightarrow x = 6 \text{ ou } x = -\frac{4}{5}$$

$$S = \left\{ -\frac{4}{5}; 6 \right\}$$

$$9) 3x^2 = 4x-1$$

$$\Leftrightarrow 3x^2 - 4x + 1 = 0$$

$$P(x) = 0 \Rightarrow \text{div. par } (x-1)$$

$$\begin{array}{c|cc|c} & 3 & -4 & 1 \\ 1 & \downarrow & & \\ \hline & 3 & -1 & 0 \end{array}$$

$$\Leftrightarrow (x-1)(3x-1) = 0$$

$$\Leftrightarrow x-1 = 0 \text{ ou } 3x-1 = 0$$

$$\Leftrightarrow x = 1 \text{ ou } x = \frac{1}{3}$$

$$4) 36x^2 - 12x + 1 = 0$$

$$\Leftrightarrow (6x-1)^2 = 0$$

$$\Leftrightarrow 6x-1 = 0$$

$$\Leftrightarrow x = \frac{1}{6}$$

$$S = \left\{ \frac{1}{6} \right\}$$

$$6) (2x-1)^2 = 49$$

$$\Leftrightarrow (2x-1)^2 - 49 = 0$$

$$\Leftrightarrow [(2x-1) - 7][(2x-1) + 7] = 0$$

$$\Leftrightarrow (2x-8)(2x+6) = 0$$

$$\Leftrightarrow 4(x-4)(x+3) = 0$$

$$\Leftrightarrow x-4 = 0 \text{ ou } x+3 = 0$$

$$\Leftrightarrow x = 4 \text{ ou } x = -3$$

$$S = \left\{ -3; 4 \right\}$$

$$8) (x-5)(3x+3) - (5-x)(2x-1) = 0$$

$$\Leftrightarrow (x-5)(3x+3) + (x-5)(2x-1) = 0$$

$$\Leftrightarrow (x-5)[(3x+3) + (2x-1)] = 0$$

$$\Leftrightarrow (x-5)(5x+2) = 0$$

$$\Leftrightarrow x-5 = 0 \text{ ou } 5x+2 = 0$$

$$\Leftrightarrow x = 5 \text{ ou } x = -\frac{2}{5}$$

$$S = \left\{ -\frac{2}{5}; 5 \right\}$$

$$S = \left\{ \frac{1}{3}; 1 \right\}$$

Chapitre 8 : les fractions rationnelles

A) Simplifie les fractions et énonce les conditions d'existence :

$$1) \frac{18x^5y^3}{6x^7y} = \frac{3y^2}{x^2}$$

C.E: $x \neq 0$ et $y \neq 0$

$$2) \frac{3a+3b}{a^2-b^2} = \frac{3(a+b)}{(a-b)(a+b)} = \frac{3}{a-b}$$

C.E: $a-b \neq 0$ et $a+b \neq 0$
 $\Leftrightarrow a \neq b$ et $a \neq -b$

$$3) \frac{ab-ac}{b^2-2bc+c^2} = \frac{a(b-c)}{(b-c)^2} = \frac{a}{b-c}$$

C.E: $b-c \neq 0$
 $\Leftrightarrow b \neq c$

$$4) \frac{18x^2+12x+2}{3x^2-5x-2} = \frac{2(9x^2+6x+1)}{(x-2)(3x+1)}$$

C.E: $x-2 \neq 0$ et $3x+1 \neq 0$
 $\Leftrightarrow x \neq 2$ et $x \neq -\frac{1}{3}$

$$P(x) = 0 \quad \left| \begin{array}{l} = \frac{2(3x+1)^2}{(x-2)(3x+1)} \\ = \frac{2(3x+1)}{x-2} \end{array} \right.$$

$$\begin{array}{c|cc|c} 3 & -5 & -2 \\ \hline 2 & \downarrow & 6 & 2 \\ \hline 3 & 1 & 0 \end{array}$$

$$5) \frac{6x^2y^2-8y^2}{9x^5-16x} = \frac{2y^2(3x^2-4)}{x(9x^4-16)}$$

$$= \frac{2y^2(3x^2-4)}{x(3x^2-4)(3x^2+4)}$$

$$= \frac{2y^2}{x(3x^2+4)}$$

C.E: $x \neq 0$ et $3x^2-4 \neq 0$ et $3x^2+4 \neq 0$
 $\Leftrightarrow x \neq 0$ et $x^2 \neq \frac{4}{3}$ et ~~$x^2 \neq \frac{4}{3}$~~
 $\Leftrightarrow x \neq 0$ et $x \neq \sqrt{\frac{4}{3}}$

$$6) \frac{4-x^2}{x^2-x-2} = \frac{(2-x)(2+x)}{(x+1)(x-2)} = \frac{-2-x}{x+1}$$

C.E: $x+1 \neq 0$ et $x-2 \neq 0$
 $\Leftrightarrow x \neq -1$ et $x \neq 2$

$$P(x) = 0$$

$$\begin{array}{c|cc|c} 1 & -1 & -2 \\ \hline -1 & \downarrow & -1 & 2 \\ \hline 1 & -2 & 0 \end{array}$$

B) Effectue les opérations:

$$1) \frac{16a^2-9b^2}{5c} \cdot \frac{3cd-7c}{4a+3b} \cdot \frac{5}{3d-7} = \frac{(4a-3b)(4a+3b) \cdot 5 \cdot (3d-7) \cdot 5}{5 \cdot 4 \cdot (4a+3b) \cdot (3d-7)} = 4a-3b$$

$$2) \frac{12ax}{3a+3b} \cdot \frac{a^2-b^2}{16a^2x^2} = \frac{\cancel{12} \cdot x \cdot (a-b)(a+b)}{\cancel{3} \cdot (a+b) \cdot \cancel{16} \cdot a^2 \cdot x^2} = \frac{a-b}{4ax}$$

$$3) \frac{4}{x-1} - \frac{3x}{x+1} - \frac{2}{x^2-1} = \frac{4}{(x-1)} - \frac{3x}{(x+1)} - \frac{2}{(x-1)(x+1)}$$

$$= \frac{4(x+1) - 3x(x-1) - 2}{(x-1)(x+1)}$$

$$= \frac{4x+4 - 3x^2+3x-2}{(x-1)(x+1)}$$

$$= \frac{-3x^2+7x+2}{(x-1)(x+1)}$$

$$4) \frac{3x}{x^2-9x+18} - \frac{2}{x^2-6x+9} = \frac{3x}{(x-3)(x-6)} - \frac{2}{(x-3)^2}$$

$P(3) = 0$

1	-9	18
3	↓	3
1	-6	0

$$= \frac{3x(x-3) - 2(x-6)}{(x-3)^2(x-6)}$$

$$= \frac{3x^2 - 9x - 2x + 12}{(x-3)^2(x-6)}$$

$$= \frac{3x^2 - 11x + 12}{(x-3)^2(x-6)}$$

$$5) \frac{a^2+2ab+b^2}{ac+bc} : \frac{a^2-b^2}{2a-2b} = \frac{(a+b)^2}{c(a+b)} \cdot \frac{2(a-b)}{(a-b)(a+b)} = \frac{2}{c}$$

$$6) \frac{2x}{2x-3} - \frac{7}{5x+4} = \frac{2x(5x+4) - 7(2x-3)}{(2x-3)(5x+4)} = \frac{10x^2+8x-14x+21}{(2x-3)(5x+4)} = \frac{10x^2-6x+21}{(2x-3)(5x+4)}$$

$$7) \frac{3x^3}{x^2-25} : \frac{12x^2}{x^2-10x+25} = \frac{\cancel{3} \cdot x^3}{(\cancel{x^2-25})(x+5)} \cdot \frac{(x-5)^2}{\cancel{12} \cdot x^2} = \frac{x(x-5)}{4(x+5)}$$

$$8) \frac{-3x}{x^2+4x+4} + \frac{2}{x^2-3x-10} = \frac{-3x}{(x+2)^2} + \frac{2}{(x+2)(x-5)}$$

$P(-2) = 0$

1	-3	-10
-2	↓	-2
1	-5	0

$$= \frac{-3x(x-5) + 2(x+2)}{(x+2)^2(x-5)}$$

$$= \frac{-3x^2+15x+2x+4}{(x+2)^2(x-5)} = \frac{-3x^2+17x+4}{(x+2)^2(x-5)}$$

$$\begin{aligned}
 9) \frac{(x-4)}{(2x-3)} - \frac{(x+7)}{(5x+4)} &= \frac{(x-4)(5x+4) - (x+7)(2x-3)}{(2x-3)(5x+4)} \\
 &= \frac{5x^2 + 4x - 20x - 16 - 2x^2 + 3x - 14x + 21}{(2x-3)(5x+4)} \\
 &= \frac{3x^2 - 27x + 5}{(2x-3)(5x+4)}
 \end{aligned}$$